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Creep damage evaluation of thick-walled spheres using a long-term creep constitutive model †

Abbas Loghman[1] and Nader Shokouhi[2]

[1]*Department of Mechanical Engineering, Faculty of Engineering, University of Kashan, Kashan, I. R. Iran*
[2]*Department of Mechanical Engineering, Sharif University of Technology, Tehran, I. R. Iran*

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Abstract

This paper describes a numerical model developed for the computation of creep damages in a thick-walled sphere subjected to an internal pressure and a thermal gradient. The model predicts the creep damage histories during the life of the sphere, owing to variations in stresses with time and through-thickness variations. The creep damage fraction is based on the Robinson's linear life fraction damage rule, which has been incorporated in a nonlinear time-dependent stress analysis. Following the stress histories, the effective stress histories are obtained and the creep damages are calculated and summed during the life of the sphere. The material long-term creep properties up to the rupture and creep rupture data are defined by the Θ projection concept [1]. The damage histories up to 38 years are calculated and the results show that the maximum damages are always located at the inner surface of the sphere, while the outer surface of the vessel sustains minimum damages.

Keywords: Creep damages; Life assessment; Theta projection; Thick-walled spheres

1. Introduction

Thick-walled spheres containing high pressure in high- temperature environments are used extensively in the areas of power generation, oil, and chemical industries. These vessels often fail by creep rupture cracking [2] and in general, failures are always catastrophic [3]. While in- service examinations provide useful information about material condition, better understanding of the sphere damage behavior is crucial before this information can be used to provide accurate predictions of the sphere future performance [4]. If an accurate time-dependent damage can be modeled for the sphere, then the sphere examinations can be scheduled in a selective manner. A major difficulty in the sphere design life or assessment of its remaining life is that stress redistribution occurs during the life of the sphere owing to time-dependency of strains. However, creep strain rates are related to the instantaneous stress condition and the material uni-axial creep constitutive model by the well-known Prandtl-Reuss equations. Improved knowledge of long-term material creep properties have shown that the creep curve shape varies significantly with changing stress and temperature [5] and the traditional assumption of constant creep rate ignores a considerable amount of information. The main objective of this paper is to evaluate the damage histories of a thick-walled sphere using a long-term material creep constitutive model defined by the Θ projection concept.

2. Material creep constitutive model

One of the key elements of an inelastic failure analysis is the material constitutive equations that describe the material deformation and rupture behavior under load. The material selected for the present

† This paper was recommended for publication in revised form by Associate Editor Youngseog Lee
*Corresponding author. Tel.: +98 361 591 2795, Fax.: +98 361 555 9930
E-mail address: aloghman@kashanu.ac.ir

Table 1. Coefficients a_i, b_i, c_i and d_i of the material constants for creep constitutive model.

	a	b	c	d
θ_1	-0.873×10^{1}	0.460×10^{-2}	-0.448×10^{-1}	0.681×10^{-4}
θ_2	-0.234×10^{-2}	0.225×10^{-1}	0.219×10^{-1}	-0.195×10^{-4}
θ_3	-0.186×10^{1}	-0.203×10^{-2}	-0.549×10^{-1}	0.799×10^{-4}
θ_4	-0.164×10^{2}	0.914×10^{-2}	-0.472×10^{-1}	0.713×10^{-4}
ε_f	-0.112×10^{1}	0.151×10^{-2}	0.547×10^{-3}	-0.472×10^{-6}

study is $\frac{1}{2}Cr,\frac{1}{2}Mo,\frac{1}{4}V$ ferritic steel. The strain-time behavior of this material has been described using the Θ projection concept as follows:

$$\varepsilon = \Theta_1(1-e^{-\Theta_2 t}) + \Theta_3(e^{\Theta_4 t}-1) \tag{1}$$

where ε is the creep strain and t is the time. Then the creep strain rate, $\dot{\varepsilon}$ is

$$\dot{\varepsilon} = \Theta_1\Theta_2 e^{-\Theta_2 t} + \Theta_3\Theta_4 e^{\Theta_4 t} \tag{2}$$

where

$$\log_{10}\Theta_i = a_i + b_i T + c_i\sigma + d_i\sigma T \quad i=1,2,3,4 \tag{3}$$

where T and σ are temperature and stress levels, respectively. Coefficients a_i, b_i, c_i and d_i are material constants. For this material, these constants are shown in table 1

The units of time temperature and stress are seconds, degree K and MPa respectively. It has also been shown that the creep fracture strains can be modeled as follows:

$$\varepsilon_f = a_i + b_i T + c_i\sigma + d_i\sigma T \qquad i=5 \tag{4}$$

On these bases Eqs. (1) and (3) can be used to construct a predicted creep curve at any stress level and temperature. The relevant rupture life is then defined as the time taken to reach the appropriate failure strain as given by Eq. (4). The full creep curves predicted for various stress levels are shown in Fig. 1 and the creep rupture contours are shown in Fig. 2.

3. Theoretical analysis

Consider a sphere with inner radius a and outer radius b, subjected to an internal pressure P_i and a temperature distribution resulting from an outward

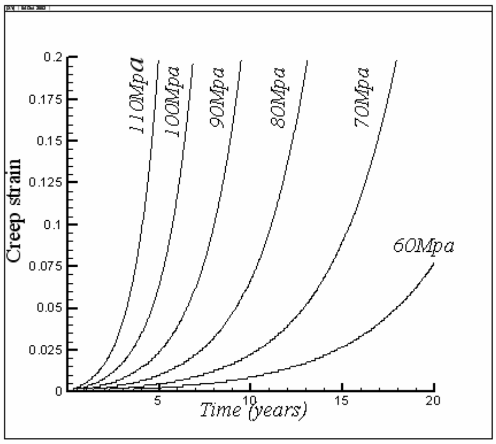

Fig. 1. Creep curves predicted by the Θ projection concept for $\frac{1}{2}Cr,\frac{1}{2}Mo,\frac{1}{4}V$ steel.

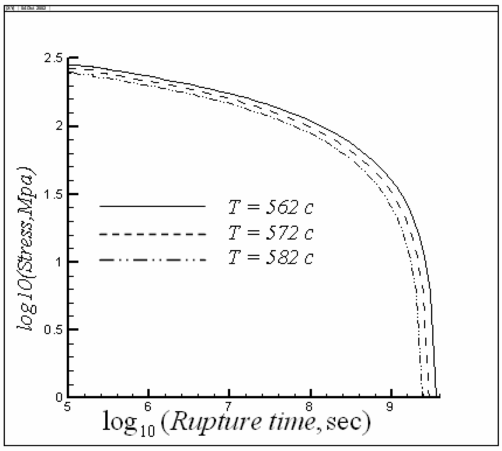

Fig. 2. Creep rupture contours predicted by Eq. (4) for $\frac{1}{2}Cr,\frac{1}{2}Mo,\frac{1}{4}V$ steel.

flow of heat owing to an inner surface temperature of T_i and outer surface temperature of T_o. Equations of equilibrium and compatibility in spherical coordinate and in dimensionless form [6] are written as:

$$\frac{dS_r}{d\rho} + \frac{2(S_r - S_\theta)}{\rho} = 0 \tag{5}$$

$$\frac{d\in_\theta}{d\rho} + \frac{\in_\theta - \in_r}{\rho} = 0 \tag{6}$$

The total strains are made up of elastic, creep, and thermal strains written in dimensionless form as:

$$\begin{aligned} \in_r &= S_r - 2\mu S_\theta + (1-\mu)\tau + \in_r^c \\ \in_\theta &= (1-\mu)S_\theta - \mu S_r + (1-\mu)\tau + \in_\theta^c \end{aligned} \tag{7}$$

where the nondimensional parameters in Eqs. (5) (7) are defined as follows:

$$\beta \equiv \frac{b}{a} \qquad \rho \equiv \frac{r}{a} \qquad P \equiv \frac{P_i}{\sigma_o} \qquad \tau \equiv \frac{E\alpha T}{(1-\mu)\sigma_o}$$
$$S_r \equiv \frac{\sigma_r}{\sigma_o} \qquad S_\theta \equiv \frac{\sigma_\theta}{\sigma_o} \qquad \in_r \equiv \frac{E\varepsilon_r}{\sigma_o} \qquad \in_\theta \equiv \frac{E\varepsilon_\theta}{\sigma_o}$$

The boundary condition of pressure and temperature at the inner and outer surfaces of the sphere are

$$r = a \quad \Rightarrow T = T_i \quad ,\sigma_r = -P_i$$
$$r = b \quad \Rightarrow T = T_o \quad ,\sigma_r = 0$$

Temperature distribution for steady-state heat conduction is as follows:

$$\tau = \frac{\beta\tau_o - \tau_i}{\beta - 1} + \frac{(\tau_i - \tau_o)\beta}{(\beta-1)\rho} \tag{8}$$

where τ_i and τ_o are dimensionless inner and outer temperatures, respectively. With the above boundary conditions and temperature distribution simultaneous solution of equilibrium, compatibility and stress-strain equations result in the following relations for the stresses [6].

$$S_r = -\frac{2}{\rho^3}\int_1^\rho \rho^2 \tau d\rho + \frac{1}{1-\mu}\int_1^\rho \frac{\in_r^c}{\rho} d\rho + \left[1 - \frac{1}{\rho^3}\right]C_1 - P \tag{9}$$

$$S_\theta = -\tau - P + \frac{1}{\rho^3}\int_1^\rho \rho^2 \tau d\rho + \left[1 + \frac{1}{2\rho^3}\right]C_1 + \frac{1}{2(1-\mu)}\in_r^c + \frac{1}{1-\mu}\int_1^\rho \frac{\in_r^c}{\rho} d\rho \tag{10}$$

where

$$C_1 = \frac{\beta^3}{\beta^3 - 1}\left[\frac{2}{\beta^3}\int_1^\beta \rho^2 \tau \, d\rho - \frac{1}{1-\mu} \times \int_1^\beta \frac{\in_r^c}{\rho} d\rho + P\right]$$

In the above equations the incompressibility condition ($\Delta\varepsilon_r^c + 2\Delta\varepsilon_\theta^c = 0$) is used and the tangential creep strains are replaced by the radial creep strain ($\Delta\varepsilon_\theta^c = -\frac{\Delta\varepsilon_r^c}{2}$).

4. Creep flow rule

Creep stresses are functions of total creep strains as shown in Eqs. (9) and (10). Creep strains are time, temperature, and stress-dependent. Therefore, increments of creep strains must be accumulated during the life of the sphere along appropriate loading path to obtain total creep strains. Creep strain rates are related to the instantaneous stress tensor and the material uniaxial creep properties by the well-known Prandtl-Reuss equations as:

$$\dot{\varepsilon}_r^c = \frac{\dot{\varepsilon}_e^c}{S_e}(S_r - S_\theta)$$
$$\dot{\varepsilon}_\theta^c = \dot{\varepsilon}_\phi^c = -\frac{\dot{\varepsilon}_r^c}{2} \tag{11}$$

where S_e is the effective stress and $\dot{\varepsilon}_e^c$ is the effective creep strain rate defined by the following equations:

$$S_e = |S_r - S_\theta|$$
$$\dot{\varepsilon}_e^c = |\dot{\varepsilon}_r^c| \tag{12}$$

The above equations and the material creep constitutive model are used in a numerical procedure, which gives the stress and strain histories. This numerical procedure is explained in detail later in this paper. The radial and tangential stress histories and the effective stress histories are obtained and shown in Figs. 3 and 4. These stress histories are used to obtain damage histories and the remaining life of the sphere. The damage histories up to 38 years are obtained and shown in Fig. 5.

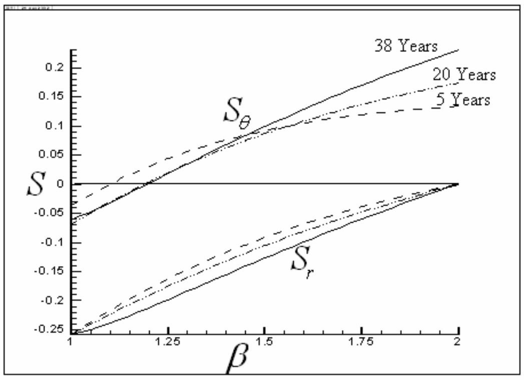

Fig. 3. Radial and tangential stress redistribution for 5, 20, and 38 years.

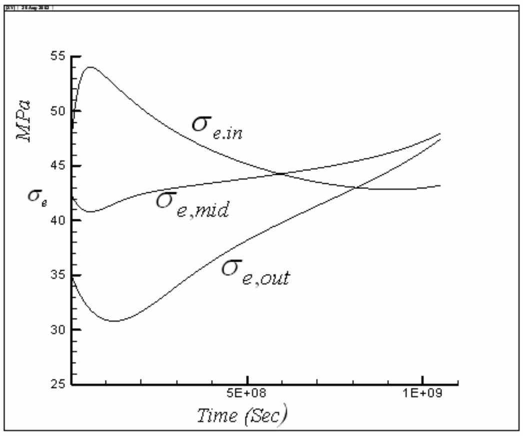

Fig. 4. Effective stress histories for the inner, middle, and outer surfaces of the sphere.

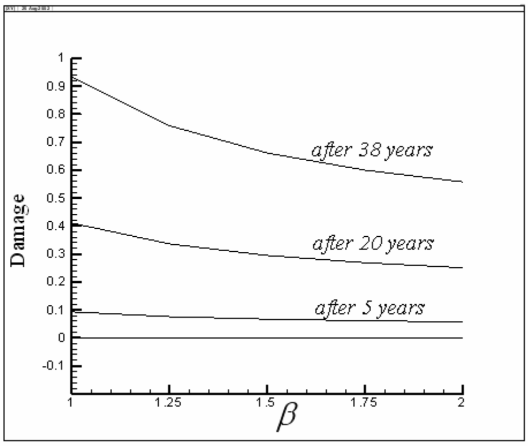

Fig. 5. History of damages versus thickness of the sphere for 5, 20, and 38 years.

5. Creep damage and remnant life

For creep rupture predictions, the Robinson's linear life-fraction damage rule is used. The damage accumulated at any point is given by the following equation:

$$D = \sum \frac{\Delta t_i}{t_r} \tag{13}$$

where Δt_i is the duration and t_r is the creep rupture time at the equivalent stress and temperature of that point. The creep rupture time is the time taken to reach the rupture strain defined by Eq. (4). Therefore, Eq. (1) may be rewritten in terms of rupture-strain and rupture time as follows

$$\varepsilon_f - \Theta_1(1 - e^{-\Theta_2 t_r}) - \Theta_3(e^{\Theta_4 t_r} - 1) = 0 \tag{14}$$

where ε_f is the rupture strain. It can be calculated for any temperature and stress level using Eq. (4). Then the rupture times can be evaluated numerically for any stress level and temperature using Eq. (14). The remaining life at any point is then given by

$$RL = (1 - D)t_r \tag{15}$$

6. Numerical procedure

The step-by-step procedure of the numerical method is written as follows:

(1) A thick-walled sphere of $\beta = 2$ is loaded with an internal pressure of $50MPa$ and a thermal gradient of $\Delta t = 20^0 C$ is considered. The temperature at the inner surface of the sphere is $550^0 C$.

(2) For the first timing step, an appropriate time increment is selected. In this solution the first time increment is $\Delta t = 10^6 Sec$. The total time is the sum of time increments as the creep process progresses in time. For the ith timing step the total time is

$$t_i = \sum_{k=1}^{i-1} \Delta t_k + \Delta t_i$$

(3) The thickness of the sphere is divided into N equal divisions. Initial values of $\Delta\varepsilon^c_{r,ij} = -0.00001$ for radial creep strain increments at all division points are assumed. These are added to the accumulated creep strains obtained from the previous timing step at all division points throughout the wall thickness of the sphere.

$$\varepsilon^c_{r,ij} = \sum_{k=1}^{i-1} \Delta\varepsilon^c_{r,kj} + \Delta\varepsilon^c_{r,ij}$$

(4) Creep strain increments in θ and ϕ directions are obtained from the symmetry and incompressibility conditions as $\Delta\varepsilon^c_{\theta,ij} = \Delta\varepsilon^c_{\phi,ij} = -\frac{1}{2}\Delta\varepsilon^c_{r,ij}$. The total creep strains in θ and ϕ directions are then

$$\varepsilon^c_{\theta,ij} = \sum_{k=1}^{i-1} \Delta\varepsilon^c_{\theta,kj} + \Delta\varepsilon^c_{\theta,ij} \qquad \varepsilon^c_{\phi,ij} = \sum_{k=1}^{i-1} \Delta\varepsilon^c_{\phi,kj} + \Delta\varepsilon^c_{\phi,ij}$$

(5) With the assumed creep strain distribution, the integrals of Eqs. (9) and (10) are evaluated. Therefore the initial estimates of creep stresses are calculated.

(6) Effective stresses are then calculated at all division points.

$$S_{e,ij} = \left| S_{r,ij} - S_{\theta,ij} \right|$$

(7) Temperature distributions are calculated using

$$\tau_j = \frac{\beta\tau_o - \tau_i}{\beta - 1} + \frac{(\tau_i - \tau_o)\beta}{(\beta - 1)\rho_j}$$

(8) With the above temperature distribution and effective stresses, creep strain rates are calculated at all division points throughout the thickness using the material's creep constitutive model Eq. (2).

$$\dot{\varepsilon}^c_{e,ij} = \Theta_{1,ij}\Theta_{2,ij}e^{-\Theta_{2,ij}t_j} + \Theta_{3,ij}\Theta_{4,ij}e^{\Theta_{,ij4}t_j}$$

where $\Theta_1, \Theta_2, \Theta_3$ and Θ_4 are temperature, and stress-dependent as follows:

$$Log\Theta_{k,ij} = a_k + b_k T_j + c_k \sigma_{e,ij} + d_k \sigma_{e,ij} T_j$$
$$k = 1, 2, 3, 4$$

(9) Radial creep strain rates are then calculated using the Prandtll-Reuss equation

$$\dot{\varepsilon}_{r,ij} = \frac{\dot{\varepsilon}^c_{e,ij}}{S_{e,ij}}(S_{r,ij} - S_{\theta,ij})$$

(10) New values for radial creep strain increments at all division points are then calculated using the above creep strain rates and the time increment as follows:

$$\Delta\varepsilon^{c,new}_{r,ij} = \dot{\varepsilon}^c_{r,ij} \times \Delta t_i$$

(11) These new obtained values of the creep strain increments are compared with the initial assumed values of the creep strain increments for the convergence of the process. If convergence has not occurred, then these new obtained values are assumed as the initial values of the creep strain increments and the procedure is repeated from step 3 until convergence is obtained at all division points throughout the thickness of the sphere.

(12) After convergence is obtained, the rupture strains are calculated at all division points using the effective stress histories and temperature distribution as follows:

$$\varepsilon_{f,ij} = a_5 + b_5 T_j + c_5 \sigma_{e,ij} + d_5 \sigma_{e,ij} T_j$$

(13) Rupture times are then calculated using the material creep constitutive equation as follows:

$$\varepsilon_{f,ij} - \theta_{1,ij}(1 - e^{-\theta_{2,ij}t_{r,ij}}) - \theta_{3,ij}(e^{\theta_{4,ij}t_{r,ij}} - 1) = 0$$

(14) Damages are then calculated at all division points and added to the previous accumulated damages as follows

$$D_{ij} = \sum \frac{\Delta t_i}{t_{r,ij}}$$

(15) The remnant life can be evaluated for all division points as follows:

$$RL_{i,j} = (1 - D_{ij}\} t_{r,ij}$$

(16) Time is advanced one increment and the procedure is repeated for the new time increment.

7. Conclusions and discussion

An improved creep constitutive equation containing full creep curve up to the rupture and the creep rupture data has been employed to obtain the damage variation with time as well as through thickness variations of damage in a thick-walled sphere. The problem contains a variable stress and a distributed temperature field. Robinson's linear life fraction damage rule has been incorporated into a nonlinear time-dependent creep stress analysis to calculate damages. For this loading condition, results show that the inner surface of the sphere is the most damaged area and the outer surface of the sphere sustains minimum damages (see Fig. 5). While minimum effective stress is located at the inner surface of the sphere later in the life of the vessel, Fig. 4, maximum damaged area is always located at the inner surface of the sphere, Fig. 5. Owing to the changes in the slope of the creep curves, a variable time increment should

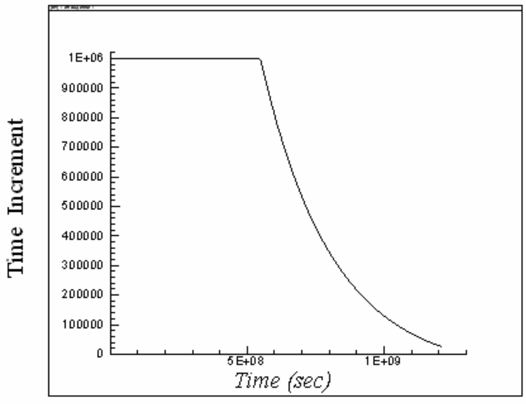

Fig. 6. Time increment variations with time for convergence of the procedure.

be employed for rapid convergence of the numerical procedure. For early stages of the creep life of the sphere where the slope of the creep curves are very small, the procedure converged with a time increment of $\Delta t = 10^6$ seconds. However, later in the life of the sphere where the slopes of the creep curves are very high, the procedure will converge with a time increment of $\Delta t = 20$ seconds. Variations in time increment with time for convergence of the procedure are shown in Fig. 6.

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Abbas Loghman received his BS degree from Sharif University of Technology, Tehran, Iran, in 1980, He then received his MS degree from the Amirkabir University of Technology, Tehran, Iran, in 1986 and his PhD degree from the University of Adelaide, South Australia, in 1995. Dr. Loghman is an Assistant Professor in the Mechanical Engineering Department of Kashan University, Kashan, Iran. His current research interests are creep and creep-fatigue life assessment of pressure vessels.

Nader Shokouhi received his BS degree from Kashan University, Iran, in 2002 and his MS degree from the Amirkabir University of Technology, Iran, in 2004. He is currently working towards his PhD degree in the Department of Mechanical Engineering, Sharif University of Technology, Iran. His current research interests are in rail vehicle dynamics modeling. He is Director of the Research and Development Department in Irankhodro Rail Transport Industries Company (IRICO).